JMO Number Theory Questions

Level: Junior   Ref No: J01   **Puzz Points: 10**

[JMO 2002 B1] A number like 4679 is called an ascending number because each digit in the number is larger than the preceding one.

    (i)   How many ascending numbers are there between 1000 and 2000?

    (ii)   How many ascending numbers are there between 1000 and 10000?

Solution: 56 and 126

Level: Junior   Ref No: J10   **Puzz Points: 13**

[JMO 2000 B4] How many different solutions are there to the letter sum on the right? Different letters stand for different digits, and no number begins with a zero.

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   J M C
+ J M O
S U M S
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Solution: 6

Level: Junior   Ref No: J11   **Puzz Points: 14**

[JMO 2000 B5] (i) Explain why the sum of three consecutive integers is always divisible by 3.

(ii) Is it true that the sum of four consecutive integers is always divisible by 4?

(iii) For which k is it true that the sum of k consecutive integers is always divisible by k?

Solution: (ii) No (iii) Whenever k is odd (Proofs obviously required)

Level: Junior   Ref No: J13   **Puzz Points: 10**

[JMO 2007 B1] Find four integers whose sum is 400 and such that the first integer is equal to twice the second integer, three times the third integer and four times the fourth integer.

Solution: 192, 96, 64, 48

Level: Junior   Ref No: J16   **Puzz Points: 13**

[JMO 2007 B4] Find a rule which predicts exactly when five consecutive integers have sum divisible by 15.

Solution: If third number is a multiple of 3.

Level: Junior   Ref No: J19   **Puzz Points: 10**

[JMO 2006 B1] In her purse, Jenny has 20 coins, with a total value of £5. There are three denominations of coin – 10p, 20p and 50p – in her purse and she has more 50p coins than 10p coins. How many of each type of coin does she have?

Solution: Two 10p, fourteen 20p, four 50p

Level: Junior   Ref No: J20   **Puzz Points: 11**

[JMO 2006 B2]
$$97 \rightarrow 63 \rightarrow 18 \rightarrow 8$$

An example of a particular type of number chain is shown above. The first number must be a positive integer. Each number after the first is the product of the digits of the previous number, so in this case 63 = 9 x 7; 18 = 6 x 3; 8 = 1 x 8. The chain stops when a single-digit number is reached.

Suppose that in such a chain the final number is 6. Find all possible two-digit first numbers for this chain.

Solution: 16, 23, 28, 32, 44, 47, 48, 61, 68, 74, 82, 84, 86

Level: Junior   Ref No: J29   **Puzz Points: 14**

[JMO 2001 B5] Observe that 49 = 4 x 9 + 4 + 9

   (i)   Find all other two-digit numbers which are equal to the product of their digits plus the sum of their digits.

(ii)   Prove that there are no three-digit numbers which are equal to the product of their digits plus the sum of their digits.

Solution: (i) 19, 29, 39, (49), 59, 69, 79, 89, 99

Level: Junior   Ref No: J31   **Puzz Points: 10**

[JMO 2011 B1] Every digit of a given positive integer is either a 3 or a 4 with each occurring at least once. The integer is divisible by both 3 and 4.

What is the smallest such integer?

Solution: 3444

Level: Junior   Ref No: J33   **Puzz Points: 12**

[JMO 2011 B3] When Dad gave out the pocket money, Amy received twice as much as her first brother, three times as much as the second, four times as much as the third and five times as much as the last brother. Peter complained that he had received 30p less than Tom.

Use this information to find all the possible amounts of money that Amy could have received.

Solution: £1.20, £1.80, £3.60, £6.00

Level: Junior   Ref No: J40   **Puzz Points: 13**

[JMO 2008 B4] For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives 3 – 4 = -1. What is the sum of all his results?

(Note: to get credit for this question, you need to give a moderately concise reason for your answer, rather than simply listing out every single 2-digit number)

Solution: 45

Level: Junior   Ref No: J50   **Puzz Points: 11**

[JMO 2010 B2] The eight-digit number "$ppppqqqq$", where p and q are digits, is a multiple of 45.

What are the possible values of *p*?

Solution: 4 and 9 (numbers are 99 990 000 and 44 445 555)

Level: Junior   Ref No: J55   **Puzz Points: 10**

[JMO 2009 B1] In 2007 Alphonse grew twice the number of grapes that Pierre did. In 2008 Pierre grew twice the number of grapes that Alphonse did. Over the two years Alphonse grew 49 000 grapes, which was 7600 less than Pierre. How many grapes did Alphonse grow in 2007?

Solution: 27 600 grapes

Level: Junior   Ref No: J60   **Puzz Points: 15**

[JMO 2009 B6] I want to choose a list of *n* different numbers from the first 20 positive integers so that no two of my numbers differ by 5. What is the largest value of *n* for which this is possible? How many different lists are there with this many numbers?

Solution: 243